



General Certificate of Education

Mathematics 6360

MFP3

Further Pure 3

Mark Scheme

2007 examination - June series

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Key to mark scheme and abbreviations used in marking

| | | | |
|--------------|--|-----|----------------------------|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| √ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| -x EE | deduct x marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

| Q | Solution | Marks | Total | Comments |
|--------------|---|---|-----------|---|
| 1(a) | $y_{PI} = kx^2 e^{5x} \Rightarrow y' = 2kxe^{5x} + 5kx^2 e^{5x}$ $\Rightarrow y'' = 2ke^{5x} + 10kxe^{5x} + 10kxe^{5x} + 25kx^2 e^{5x}$ $\Rightarrow 2ke^{5x} + 20kxe^{5x} + 25kx^2 e^{5x}$ $-10(2kxe^{5x} + 5kx^2 e^{5x}) + 25kx^2 e^{5x} = 6e^{5x}$ | M1 A1 A1ft M1 A1 | 6 | Product rule to differentiate $x^2 e^{5x}$ Substitution into differential equation |
| | $2k = 6 \Rightarrow k = 3$ | A1ft | | Only fit if xe^{5x} and $x^2 e^{5x}$ terms all cancel out |
| (b) | Aux. eqn. $m^2 - 10m + 25 = 0 \Rightarrow m = 5$ | B1 | 4 | PI |
| | CF is $(A + Bx)e^{5x}$ GS $y = (A + Bx)e^{5x} + 3x^2 e^{5x}$ | M1 M1 A1ft | | Their CF + their/our PI ft only on wrong value of k |
| Total | | | 10 | |
| 2(a) | $y_1 = 2 + 0.1 \times \sqrt{1^2 + 2^2 + 3}$ $y(1.1) = 2 + 0.1 \times \sqrt{8}$ $y(1.1) = 2.28284\dots = 2.2828$ to 4dp | M1 A1 A1 | 3 | |
| | (b) | $k_1 = 0.1 \times \sqrt{8} = 0.2828$ $k_2 = 0.1 \times f(1.1, 2.2828\dots)$ $= 0.1 \times \sqrt{9.42137\dots} = 0.3069(425\dots)$ $y(1.1) = y(1) + \frac{1}{2}[0.28284\dots + 0.30694\dots]$ $2.29489\dots = 2.2949$ to 4dp | | M1 A1ft M1 A1 m1 A1 |
| Total | | | 9 | |
| 3 | IF is $e^{\int \tan x dx}$ $= e^{-\ln \cos x} = e^{\ln \sec x}$ $= \sec x$ $\frac{d}{dx}(y \sec x) = \sec^2 x$ $y \sec x = \int \sec^2 x dx$ $y \sec x = \tan x + c$ $y = 3$ when $x = 0 \Rightarrow 3 \sec 0 = 0 + c$ $c = 3 \Rightarrow y \sec x = \tan x + 3$ | M1 A1 A1ft M1A1 A1 m1 A1 | 8 | Accept either ft on earlier sign error Condone missing c OE; condone solution finishing at $c = 3$ provided no errors |
| | Total | | | |

MFP3 (cont)

| Q | Solution | Marks | Total | Comments |
|--------|--|--|------------|---|
| 4(a) | $(\cos \theta + \sin \theta)^2 = \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta$ $= 1 + \sin 2\theta$ | B1 | 1 | AG (be convinced) |
| (b) | $(x^2 + y^2)^3 = (x + y)^4$ $(r^2)^3 = (r \cos \theta + r \sin \theta)^4$ $r^6 = r^4 (\cos \theta + \sin \theta)^4$ $r^6 = r^4 (1 + \sin 2\theta)^2$ $r^2 = (1 + \sin 2\theta)^2$ $\Rightarrow r = (1 + \sin 2\theta) \{r \geq 0\}$ | M2,1,0 M1 | | [M1 for one of $x^2 + y^2 = r^2$ OE, $x = r \cos \theta, y = r \sin \theta$ used] Uses (a) OE at any stage |
| (c)(i) | $r = 0 \Rightarrow \sin 2\theta = -1$ $2\theta = \sin^{-1}(-1); = -\frac{\pi}{2}, \frac{3\pi}{2}$ $\theta = -\frac{\pi}{4}; \frac{3\pi}{4}$ | A1 M1 A1A1ft | 4 3 | CSO; AG A1 for either |
| (ii) | Area = $\frac{1}{2} \int (1 + \sin 2\theta)^2 d\theta$ $= \frac{1}{2} \int (1 + 2\sin 2\theta + \sin^2 2\theta) d\theta$ $= \frac{1}{2} \int \left(1 + 2\sin 2\theta + \frac{1}{2}(1 - \cos 4\theta) \right) d\theta$ $= \left[\frac{3}{4}\theta - \frac{1}{2}\cos 2\theta - \frac{1}{16}\sin 4\theta \right]$ $= \left[\frac{3}{4}\theta - \frac{1}{2}\cos 2\theta - \frac{1}{16}\sin 4\theta \right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$ $= \left(\frac{9\pi}{16} \right) - \left(-\frac{3\pi}{16} \right)$ $= \frac{3\pi}{4}$ | M1 B1 M1 A1ft m1 A1 | 6 | Use of $\frac{1}{2} \int r^2 d\theta$ Correct expansion of $(1 + \sin 2\theta)^2$ Attempt to write $\sin^2 2\theta$ in terms of $\cos 4\theta$ Correct integration ft wrong coefficients only Using c's values from (c)(i) as limits or the correct limits CSO |
| | Total | | 14 | |

MFP3 (cont)

| Q | Solution | Marks | Total | Comments |
|-------------|---|--------------|--------------|--|
| 5(a) | $u = \frac{dy}{dx} + x \Rightarrow \frac{du}{dx} = \frac{d^2y}{dx^2} + 1$ | M1A1 | | |
| | $(x^2 - 1)\left(\frac{du}{dx} - 1\right) - 2x(u - x) = x^2 + 1$ | M1 | | Substitution into LHS of DE as far as no ys |
| | DE $\Rightarrow (x^2 - 1)\frac{du}{dx} - 2xu = 0$ | | | |
| | $\Rightarrow \frac{du}{dx} = \frac{2xu}{x^2 - 1}$ | A1 | 4 | CSO; AG |
| (b) | $\int \frac{1}{u} du = \int \frac{2x}{x^2 - 1} dx$ | M1 A1 | | Separate variables |
| | $\ln u = \ln x^2 - 1 + \ln A$ | A1A1 | | |
| | $u = A(x^2 - 1)$ | A1 | 5 | |
| (c) | $\frac{dy}{dx} + x = A(x^2 - 1)$ | M1 | | Use (b) ($\neq 0$) to form DE in y and x |
| | $\frac{dy}{dx} = A(x^2 - 1) - x$ | | | |
| | $y = A\left(\frac{x^3}{3} - x\right) - \frac{x^2}{2} + B$ | M1 | | Solution must have two different constants and correct method used to solve the DE |
| | | A1ft | 3 | |
| | Total | | 12 | |

MFP3 (cont)

| Q | Solution | Marks | Total | Comments |
|---|--|----------------------------|-----------|--|
| 6(a)(i) | $f(x) = \ln(1 + e^x)$: $f(0) = \ln 2$ $f'(x) = \frac{e^x}{1+e^x} \quad f'(0) = \frac{1}{2}$ $f''(x) = \frac{(1+e^x)e^x - e^x e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$ $f''(0) = \frac{1}{4}$ so first three terms are: $f(x) = \ln 2 + \frac{1}{2}x + \frac{1}{4} \frac{x^2}{2!} = \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2$ | B1 M1 A1 M1 A1 | | Chain rule Quotient rule OE |
| (ii) | $f'''(x) = \frac{(1+e^x)^2 e^x - e^x [2(1+e^x)e^x]}{(1+e^x)^4}$ $f'''(0) = \frac{4-4}{2^4} = 0$ {so coefficient of x^3 is zero} | M1 A1ft A1 | 6 3 | Chain rule with quotient/product rule ft on $f''(x) = ke^x(1+e^x)^n$ (integer $n < 0$) CSO; AG; All previous differentiation correct |
| SC for those not using Maclaurin's theorem: maximum of 4/9 | | | | |
| (b) | $\frac{1}{2}x + \frac{1}{8}x^2$ | B1 | 1 | |
| (c) | $\ln\left(1 - \frac{x}{2}\right) =$ $\left(-\frac{x}{2}\right) - \frac{1}{2}\left(-\frac{x}{2}\right)^2 + \frac{1}{3}\left(-\frac{x}{2}\right)^3 - \dots$ | B1 | 1 | |
| (d) | $\ln\left(\frac{1+e^x}{2}\right) + \ln\left(1 - \frac{x}{2}\right) = -\frac{x^3}{24} + \dots$ $x - \sin x \approx x - \left[x - \frac{x^3}{3!} + \dots\right] \approx \frac{x^3}{3!} + \dots$ $\left[\frac{\ln\left(\frac{1+e^x}{2}\right) + \ln\left(1 - \frac{x}{2}\right)}{x - \sin x} \right] = \frac{-\frac{1}{24}x^3 + \dots}{\frac{1}{6}x^3 + o(x^5)}$ $= \frac{-\frac{1}{24}x^3 + \dots}{x^3 \left[\frac{1}{6} + o(x^2)\right]} = \frac{-\frac{1}{24} + \dots}{\frac{1}{6} + o(x^2)}$ $\lim_{x \rightarrow 0} \dots = -\frac{1}{4}$ | M1 B1 M1 | | Uses previous expansions to obtain first non-zero term of the form kx^3 |
| Total | | | 15 | CSO |

MFP3 (cont)

| Q | Solution | Marks | Total | Comments |
|-------------|--|--------------|--------------|--------------------------------------|
| 7(a) | 0 | B1 | 1 | |
| (b) | $u = xe^{-x} + 1 \Rightarrow du = (e^{-x} - xe^{-x}) dx$ $\int \frac{e^{-x}(1-x)}{xe^{-x} + 1} dx = \int \frac{1}{u} du = \ln u + c$ $= \ln(xe^{-x} + 1) \{+ c\}$ | M1 | | Attempts to find du |
| (c) | $\int \frac{1-x}{x+e^x} dx = \int \frac{e^{-x}(1-x)}{xe^{-x} + 1} dx$ $\int_1^\infty \frac{1-x}{x+e^x} dx = \lim_{a \rightarrow \infty} [\ln(xe^{-x} + 1)]_1^a$ $= \lim_{a \rightarrow \infty} \{ \ln(ae^{-a} + 1) \} - \ln(e^{-1} + 1)$ $= \ln \left\{ \lim_{a \rightarrow \infty} (ae^{-a} + 1) \right\} - \ln(e^{-1} + 1)$ $= \ln 1 - \ln(e^{-1} + 1) = -\ln(e^{-1} + 1)$ | B1 | 2 | Condone missing c |
| | | M1 | | For using part (b) and $F(B) - F(A)$ |
| | | M1 A1 | 4 | For using limiting process |
| | Total | | 7 | |
| | TOTAL | | 75 | |